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MATHEMATICS 20-1

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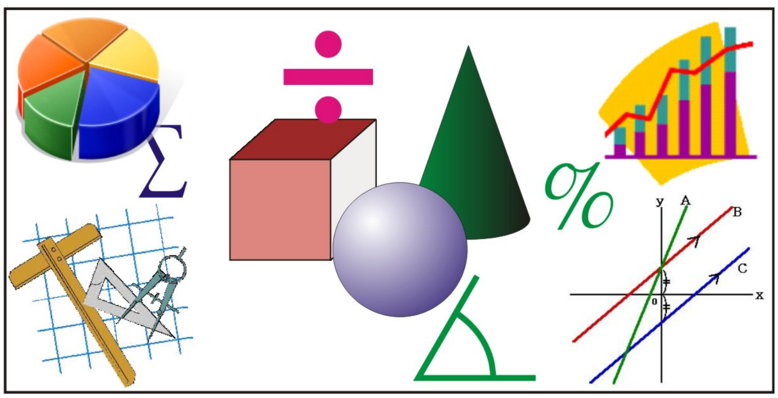
STUDENT GUIDE AND RESOURCE BOOK



Key to Student Success
with Grade 11 Mathematics

One of a series of publications by Raven Research Associates for Secondary and Elementary Mathematics

Alan R. Taylor, Ed. D.
Bill Kokoskin, M.A.

Raven Research Associates, Inc.

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7.4 The Discriminant

- In the section on graphing quadratic equations we showed the following examples where the graph intersected the x-axis in two locations, in one location, and in no locations.

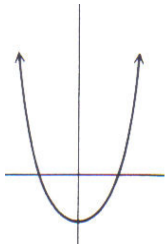

Figure 1

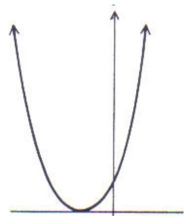

Figure 2

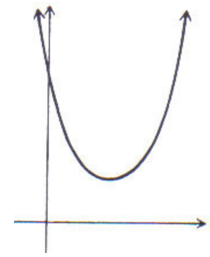

Figure 3

- In Figure 1 the equation has 2 real roots, in Figure 2 it has one double real root, and in Figure 3 it has no real roots (we could say it has 2 imaginary roots).
- We can tell what types of roots a quadratic equation has by taking a closer look at the quadratic formula below.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Discriminant

- The expression $\mathbf{b^2 - 4ac}$, which is under the radical sign in the quadratic formula, is called the **Discriminant (D)**. By looking at its value we can tell certain things about the roots of the equation. Some examples follow.

Equation	**Solution Using the Formula**	**Value of Disriminant and Nature of the roots**
Example 1 $4x^2 - 12x + 9 = 0$	$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \bullet 4 \bullet 9}}{2 \bullet 4}$ $= \frac{12 \pm \sqrt{144 - 144}}{8} = \frac{12 \pm \sqrt{0}}{8}$ $= \frac{12}{8} = \frac{3}{2}$	*$b^2 - 4ac = 0$* *$D = 0$* *The root is $\frac{3}{2}$* *The roots are real and equal.*
Example 2 $2x^2 - x - 6 = 0$	$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \bullet 2 \bullet (-6)}}{2 \bullet 2}$ $x = \frac{1 \pm \sqrt{1 - (-48)}}{4} = \frac{1 \pm \sqrt{49}}{4}$ $x = \frac{1+7}{4}, \frac{1-7}{4}$ $x = 2, -\frac{3}{2}$	*$D = 49$* *$b^2 - 4ac > 0$* *The roots are 3 and $-\frac{3}{2}$* *The two roots are real and unequal*
Example 3 $x^2 + x + 3 = 0$	$x = \frac{-1 \pm \sqrt{1^2 - 4 \bullet 1 \bullet 3}}{2 \bullet 1}$ $x = \frac{-1 \pm \sqrt{1 - 12}}{2} = \frac{-1 \pm \sqrt{-11}}{2}$	*$D = -11$* *$b^2 - 4ac < 0$* *We can't find the square root of a negative number in the real number system.* *There are no real roots.*

Nature of the Roots of a Quadratic Equation

- Next we will make the following generalizations about the value of the discriminant ($D = b^2 - 4ac$) and the nature of the roots of a quadratic equation.

For the quadratic equation $ax^2 + bx + c$, $a \neq 0$			
$D = b^2 - 4ac > 0$	▪ Roots are real and unequal	If a, b, and c are rational and $D \geq 0$	
$D = b^2 - 4ac = 0$	▪ Roots are real and equal	When $b^2 - 4ac$ is a perfect square	▪ The roots are rational
$D = b^2 - 4ac < 0$	▪ There are no real roots	When $b^2 - 4ac$ is a not a perfect square	▪ The roots are irrational

e.g. Find the value of each discriminant and describe the nature of the roots.

1. $x^2 - 6x - 3 = 0$
   - *$D = b^2 - 4ac$*
   - *$D = (-6)^2 - 4(1)(-3) = 36 + 12 = 48$*
   - *$D > 0$, so there are 2 real and unequal roots*
   - *Since a, b, c are rational and D is not a perfect square the roots are irrational*

2. $2x^2 + x + 4 = 0$
   - *$D = b^2 - 4ac$*
   - *$D = (1)^2 - 4(2)(4) = 1 - 32 = -31$*
   - *$D < 0$, so there are no real roots*

3. $9x^2 - 6x + 1 = 0$
   - *$D = b^2 - 4ac$*
   - *$D = (-6)^2 - 4(9)(1) = 36 - 36 = 0$*
   - *$D = 0$, so there are 2 real and equal roots(a double root)*
   - *Since a, b, c are rational and D is a perfect square the roots are rational*

4. $2x^2 - \sqrt{3}\,x - 5 = 0$
   - *$D = b^2 - 4ac$*
   - *$D = (-\sqrt{3}\,)^2 - 4(2)(-5) = 3 + 40 = 43$*
   - *$D > 0$, so there are 2 real and unequal roots*
   - *Since D is not a perfect square the roots are irrational*

5. $2x - \frac{1}{3}x^2 = 6$
   - *$D = b^2 - 4ac$*
   - *$D = (2)^2 - 4(-\frac{1}{3})(-6) = 4 - 8 = -4$*
   - *$D < 0$, so there are no real roots*

Exercises 7.4 The Discriminant

1. Find the value of the discriminant for each equation and give the nature of its roots.

a. $3x^2 - x - 1 = 0$

b. $-2x^2 + x + 1 = 0$

c. $1-5x^2 = 8$

d. $-3x^2 - x -5 = 0$

e. $2x(1 - 2x) = x - 5$

f. $\sqrt{3}x^2 - \sqrt{5}x + \sqrt{3} = 0$

g. $3t^2 - \sqrt{5}\,t = 7$

h. $\sqrt{2}x^2 - 3x - \sqrt{8} = 0$

i. $\dfrac{1}{x+1} = 2 + 3x$

j. $\dfrac{r-3}{r} = \dfrac{2r+1}{3}$

2. Determine all values of k for which each equation will have the indicated number of roots.

a. $x^2 + kx + 2$; 1 real double root

b. $x^2 - kx + 2$; no real roots

c. $2x^2 - 3x + k$: 2 real unequal roots

d. $-3x^2 - kx - 5 = 0$; 1 real double root

3. In each of the following determine k to that (i) there are two real roots, (ii) there is one double real root, and (iii) there are no real roots.

a. $x^2 - kx + 5 = 0$

(i) there are two real roots ______

(ii) there is one double real root ______

(iii) there are no real roots ______

b. $kx^2 + 4x + 2 = 0$

(i) there are two real roots ______

(ii) there is one double real root ______

(iii) there are no real roots ______

c. $x^2 + 2x = 1 - 2k$

(i) there are two real roots ______

(ii) there is one double real root ______

(iii) there are no real roots ______

d. $2x^2 + kx + k = 0$

(i) there are two real roots ______

(ii) there is one double real root ______

(iii) there are no real roots ______

4. For what value(s) of k would the following equation have one double real root?

a. $kx^2 - x + 2 = 0$

b. $x^2 - kx + 2 = 0$