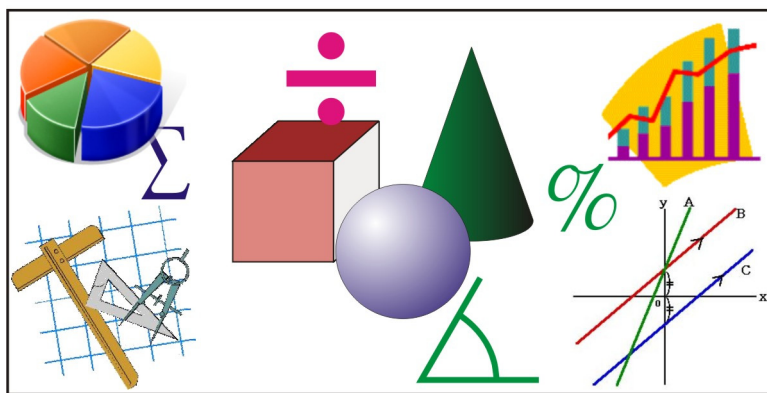


New!

**RAVEN'S GUIDE TO
ALBERTA
MATHEMATICS 20-1**

**LINKED DIRECTLY TO NEW CURRICULUM REQUIREMENTS
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**STUDENT GUIDE AND
RESOURCE BOOK**



**Key to Student Success
with Grade 11 Mathematics**

**One of a series of publications by Raven Research Associates
for Secondary and Elementary Mathematics**

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Table of Contents

	Page		Page
Chapter 1 - Absolute Value of Numbers and Introduction to Radicals			
1.1 Absolute Value of a Number and the Number Line	2	5.4 Factoring $a[f(x)]^2 + b(f(x)) + c, a \neq 0$	151
1.2 Equations and Inequalities Involving Absolute Value	7	5.5 Factoring $a^2[f(x)]^2 - b^2[g(y)]^2; a \neq 0, b \neq 0$	153
1.3 Powers and Roots of Numbers	14	5.6 Combination of Factoring	155
1.4 Ordering Radicals and Using a Calculator to Approximate Values	18	5.7 Factor Theorem	157
1.5 Simplifying Radicals by Factoring	22	Chapter 6 - Relations and Quadratic Functions	
1.6 Adding and Subtracting Radicals	25	6.1 Review of Relations and Functions	161
1.7 Multiplication and Division of Square Root Radicals	30	6.2 Graphs of Quadratic Functions	172
Chapter 2- Properties and Applications of Radicals		6.3 Transformations of Quadratic Functions	186
2.1 Writing Radicals in Simplest Form	42	6.4 Reciprocal Functions	191
2.2 Product of a Binomial times a Binomial	46	6.5 Graphing the Absolute Value Function	197
2.3 Conjugates of Binomials and Rationalizing Denominators	50	6.6 Solving Absolute-Value Equations Algebraically and Graphically	205
2.4 Relationships between Roots, Absolute Values and Signs	53	Chapter 7 - Applications with Quadratic Functions	
2.5 Solving Equations Involving Radicals	55	7.1 Completing the Square	214
2.6 Problems Involving Radical Expressions	59	7.2 Maximum and Minimum Problems	219
Chapter 3 - Rational Expressions and Equations		7.3 Solving Quadratic Equations	224
3.1 Rational Numbers (Review)	69	7.3.1 Solving by graphing	224
3.2 Addition and Subtraction of Fractions (Review)	73	7.3.2 Solving by factoring	229
3.3 Multiplication and Division of Fractions (Review)	77	7.3.3 Solving by completing the square	233
3.4 Rational Expressions	82	7.3.4 The quadratic formula	235
3.5 Adding and Subtracting Rational Expressions	85	7.4 The Discriminant	239
3.6 Multiplying and Dividing Rational Expressions	87	Chapter 8 - Sequences and Series	
3.7 Multiple Operations with Rational Expressions	90	8.1 Arithmetic Sequences	246
3.8 Rational Equations	92	8.2 Arithmetic Series	252
3.9 Solving Problems Involving Rational Equations	95	8.3 Geometric Sequences	257
Chapter 4 - Trigonometry		8.4 Geometric Series	261
4.1 Definition of Trig Functions and Angles in Standard Position	103	8.5 Sums of Infinite Geometric Series	265
4.2 Special Angles	114	Chapter 9 - Inequalities	
4.3 Law of Sines	120	9.1 Graphing Inequalities in One Variable in Two Dimensions	276
4.4 Law of Cosines	125	9.2 Graphing Inequalities in Two Variables	279
4.5 Solving General Triangles (Ambiguous Case)	129	9.3 Graphing Systems of Linear and Quadratic Inequalities	284
Chapter 5 - Factoring Polynomials		9.4 Graphing Quadratic Inequalities in One Variable	290
5.1 Review of Factoring in General	140	9.5 Problems for Quadratic Inequalities	295
5.2 Factoring $ax^2 + bx + c, a \neq 0$	146	Chapter 10 - Linear and Quadratic Systems	
5.3 Factoring $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$	149	10.1 Linear-Quadratic Systems	301
		10.2 Quadratic-Quadratic Systems	304
		10.3 Problems for Systems	307

7.4 The Discriminant

- In the section on graphing quadratic equations we showed the following examples where the graph intersected the x-axis in two locations, in one location, and in no locations.

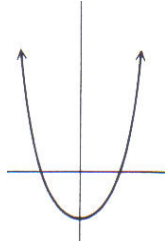


Figure 1

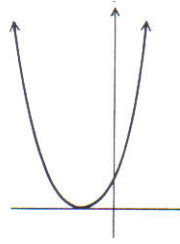


Figure 2

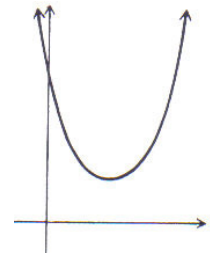


Figure 3

- In Figure 1 the equation has 2 real roots, in Figure 2 it has one double real root, and in Figure 3 it has no real roots (we could say it has 2 imaginary roots).
- We can tell what types of roots a quadratic equation has by taking a closer look at the quadratic formula below.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The Discriminant

- The expression $b^2 - 4ac$, which is under the radical sign in the quadratic formula, is called the **Discriminant (D)**. By looking at its value we can tell certain things about the roots of the equation. Some examples follow.

Equation	Solution Using the Formula	Value of Discriminant and Nature of the roots
<p>Example 1</p> $4x^2 - 12x + 9 = 0$	$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot 4 \cdot 9}}{2 \cdot 4}$ $= \frac{12 \pm \sqrt{144 - 144}}{8} = \frac{12 \pm \sqrt{0}}{8}$ $= \frac{12}{8} = \frac{3}{2}$	$b^2 - 4ac = 0$ $D = 0$ <p>The root is $\frac{3}{2}$</p> <p>The roots are real and equal.</p>
<p>Example 2</p> $2x^2 - x - 6 = 0$	$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-6)}}{2 \cdot 2}$ $x = \frac{1 \pm \sqrt{1 - (-48)}}{4} = \frac{1 \pm \sqrt{49}}{4}$ $x = \frac{1+7}{4}, \frac{1-7}{4}$ $x = 2, -\frac{3}{2}$	$D = 49$ $b^2 - 4ac > 0$ <p>The roots are 3 and $-\frac{3}{2}$</p> <p>The two roots are real and unequal</p>
<p>Example 3</p> $x^2 + x + 3 = 0$	$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$ $x = \frac{-1 \pm \sqrt{1 - 12}}{2} = \frac{-1 \pm \sqrt{-11}}{2}$	$D = -11$ $b^2 - 4ac < 0$ <p>We can't find the square root of a negative number in the real number system.</p> <p>There are no real roots.</p>

Nature of the Roots of a Quadratic Equation

- Next we will make the following generalizations about the value of the discriminant ($D = b^2 - 4ac$) and the nature of the roots of a quadratic equation.

For the quadratic equation $ax^2 + bx + c$, $a \neq 0$		
$D = b^2 - 4ac > 0$	▪ Roots are real and unequal	If a , b , and c are rational and $D \geq 0$ When $b^2 - 4ac$ is a perfect square When $b^2 - 4ac$ is not a perfect square
$D = b^2 - 4ac = 0$	▪ Roots are real and equal	
$D = b^2 - 4ac < 0$	▪ There are no real roots	
		▪ The roots are rational
		▪ The roots are irrational

e.g. Find the value of each discriminant and describe the nature of the roots.

- $x^2 - 6x - 3 = 0$
 - $D = b^2 - 4ac$
 - $D = (-6)^2 - 4(1)(-3) = 36 + 12 = 48$
 - $D > 0$, so there are 2 real and unequal roots
 - Since a , b , c are rational and D is not a perfect square the roots are irrational
- $2x^2 + x + 4 = 0$
 - $D = b^2 - 4ac$
 - $D = (1)^2 - 4(2)(4) = 1 - 32 = -31$
 - $D < 0$, so there are no real roots
- $9x^2 - 6x + 1 = 0$
 - $D = b^2 - 4ac$
 - $D = (-6)^2 - 4(9)(1) = 36 - 36 = 0$
 - $D = 0$, so there are 2 real and equal roots (a double root)
 - Since a , b , c are rational and D is a perfect square the roots are rational
- $2x^2 - \sqrt{3}x - 5 = 0$
 - $D = b^2 - 4ac$
 - $D = (-\sqrt{3})^2 - 4(2)(-5) = 3 + 40 = 43$
 - $D > 0$, so there are 2 real and unequal roots
 - Since D is not a perfect square the roots are irrational
- $2x - \frac{1}{3}x^2 = 6$
 - $D = b^2 - 4ac$
 - $D = (2)^2 - 4(-\frac{1}{3})(-6) = 4 - 8 = -4$
 - $D < 0$, so there are no real roots

Exercises 7.4 The Discriminant

1. Find the value of the discriminant for each equation and give the nature of its roots.

a. $3x^2 - x - 1 = 0$

b. $-2x^2 + x + 1 = 0$

c. $1 - 5x^2 = 8$

d. $-3x^2 - x - 5 = 0$

e. $2x(1 - 2x) = x - 5$

f. $\sqrt{3}x^2 - \sqrt{5}x + \sqrt{3} = 0$

g. $3t^2 - \sqrt{5}t = 7$

h. $\sqrt{2}x^2 - 3x - \sqrt{8} = 0$

i. $\frac{1}{x+1} = 2 + 3x$

j. $\frac{r-3}{r} = \frac{2r+1}{3}$

2. Determine all values of k for which each equation will have the indicated number of roots.

a. $x^2 + kx + 2$; 1 real double root

b. $x^2 - kx + 2$; no real roots

c. $2x^2 - 3x + k$; 2 real unequal roots

d. $-3x^2 - kx - 5 = 0$; 1 real double root

3. In each of the following determine k to that (i) there are two real roots, (ii) there is one double real root, and (iii) there are no real roots.

a. $x^2 - kx + 5 = 0$

(i) there are two real roots _____

(ii) there is one double real root _____

(iii) there are no real roots _____

b. $kx^2 + 4x + 2 = 0$

(i) there are two real roots _____

(ii) there is one double real root _____

(iii) there are no real roots _____

c. $x^2 + 2x = 1 - 2k$

(i) there are two real roots _____

(ii) there is one double real root _____

(iii) there are no real roots _____

d. $2x^2 + kx + k = 0$

(i) there are two real roots _____

(ii) there is one double real root _____

(iii) there are no real roots _____

4. For what value(s) of k would the following equation have one double real root?

a. $kx^2 - x + 2 = 0$

b. $x^2 - kx + 2 = 0$