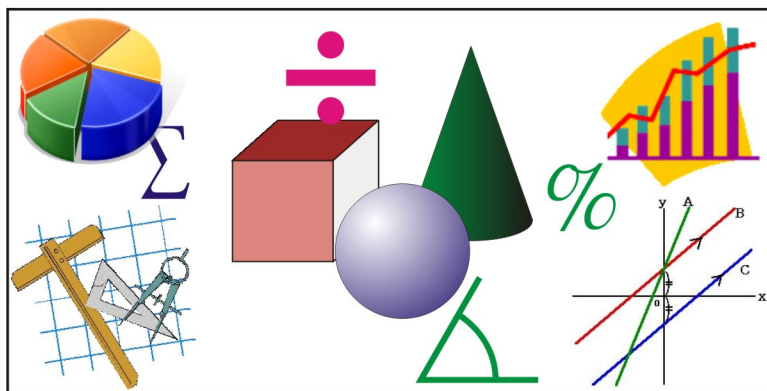


RAVEN'S LITERACY FOUNDATIONS LEVEL 6 MATHEMATICS

UP-GRADING PROGRAM

(Designed for the Western Provinces and the Territories)

STUDENT GUIDE AND RESOURCE BOOK



The Key to Student Success

One of a series of publications by Raven Research Associates
for Secondary and Elementary Mathematics

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Introduction

This book is intended to provide students taking **Literacy Foundations Level 6 Mathematics** with a practical resource designed to enhance success. It is linked to the provincial mathematics curriculum and designed by experienced teachers of mathematics to provide students with greater success in these courses. It includes the following features:

- Clear Descriptions of the Key Concepts
- Numerous Examples with Step-by-Step Solutions
- Many Practice Exercises to Reinforce Understanding and Application
- Review Exercises with a Range of Difficulty Levels
- All Answers which are listed at the Back of the Book
- Produced by Experienced Teachers of Mathematics
- Hand in Exercises
- Attractively Bound and Formatted for Clarity and Ease of Access

The content areas listed in the Table of Contents, shown in the next page, are linked to the following Prescribed Learning Outcomes for Literacy Foundations Level 6 Mathematics.

NUMBER

<p>A1 demonstrate an understanding of perfect square and square root concretely, pictorially, and symbolically</p> <p>A2 determine the square root of positive whole and rational numbers that are perfect squares</p> <p>A3 determine, using technology, the approximate square root of positive rational numbers that are non-perfect squares and justify their reasonableness</p> <p>A4 demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:</p>	<ul style="list-style-type: none"> - representing repeated multiplication using powers - using patterns to show that a power with an exponent of zero is equal to one - solving problems involving powers <p>A5 demonstrate an understanding of rational numbers by</p> <ul style="list-style-type: none"> - comparing and ordering rational numbers - solving problems that involve arithmetic operations on rational numbers, with or without technology <p>A6 explain and apply the order of operations, including exponents, with or without technology</p>
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PATTERNS AND RELATIONS

<p>Patterns</p> <p>B1 determine if the relationship between two variables is linear and justify the reasoning</p> <p>B2 generate a pattern from a problem using linear equations and verify by substitution</p> <p>B3 graph linear relations, analyse the graph, and interpolate or extrapolate from the graph to solve problems</p> <p>Variables and Equations</p> <p><i>It is expected that students will:</i></p> <p>B4 model and solve problems using linear equations of the form</p> <ul style="list-style-type: none"> - $a\chi = b$, $\chi/a = b$, $a \neq 0$, $a\chi + b = c$ - $\chi/a + b = d$, $a \neq 0$, $a(\chi + b) = c$, $a\chi + b = c\chi + d$ - $a(b\chi + c) = d(e\chi + f)$, $- a\chi = b$, $\chi \neq 0$ <p>where a, b, c, d, e, and f are rational numbers</p>	<p>B5 solve single variable linear inequalities with rational coefficients</p> <p>B6 demonstrate an understanding of polynomials (of degree less than or equal to 2) by</p> <ul style="list-style-type: none"> - identifying the variables, degree, number of terms and coefficients, including the constant term of a given simplified polynomial expression - describing a situation for a given first-degree polynomial expression - matching equivalent polynomial expressions given in simplified form (e.g., $4x - 3x^2 + 2$ is equivalent to $-3x^2 + 4x + 2$) <p>B7 add and subtract polynomial expressions (of degree less than or equal to 2)</p> <p>B8 multiply and divide polynomial expressions by monomials (of degree less than or equal to 2)</p>
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SHAPE AND SPACE

<p>Measurement</p> <p>C1 develop and apply the Pythagorean theorem to solve problems</p> <p>3-D Objects and 2-D Shapes</p> <p>C2 explain and describe polygons and polyhedra in terms of their edges, faces, and vertices</p>	<p>C3 determine the surface area of:</p> <ul style="list-style-type: none"> - right rectangular prisms - right triangular prisms - right cylinders - composite 3-D objects to solve problems
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SAMPLE EXERCISE

1.5.2 Evaluating Exponential Expressions with Numerical Bases

- As discussed previously, a number multiplied by itself several times can be shown as a base written to an exponent.
e.g. (i) $3 \times 3 \times 3 \times 3 = 81$ can be shown as $3^4 = 81$ where 3 is called the **base** and 4 the **power or exponent**
(ii) $4 \times 4 \times 4 \times 4 = 4^5$ where 4 is the base and 5 is the exponent
- A number that is multiplied by itself is called the **base**
- The number of times it is multiplied by itself is called the **power or exponent**.

Only non-zero numbers will be written as bases to powers or exponents.

e.g. (i) $(0.3)^4 = (0.3)(0.3)(0.3)(0.3)$

(ii) $\left(\frac{1}{3}\right)^3 = \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$

Examples with Solutions

Write each product as a base and an exponent.

1. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

Solution:

- The number 2 is multiplied by itself 6 times
- As a result the base is 2 and the exponent is 6
- $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

2. $0.3 \times 0.3 \times 0.3$

Solution:

- The number 0.3 is multiplied by itself 3 times
- As a result the base is 0.3 and the exponent is 3
- $0.3 \times 0.3 \times 0.3 = (0.3)^3$

3. $1.5 \times 1.5 \times 1.5 \times 1.5 \times 1.5$

Solution:

- The number 1.5 is multiplied by itself 5 times
- As a result the base is 1.5 and the exponent is 5
- $1.5 \times 1.5 \times 1.5 \times 1.5 \times 1.5 = (1.5)^5$

4. $3 \times 3 \times 3 \times 0.2 \times 0.2 \times 0.2 \times 0.2$

Solution:

- The number 3 is multiplied by itself 3 times and the number 0.2 is multiplied by itself 4 times
- As a result the base 3 has an exponent of 3 and the base 0.2 has an exponent of 4
- $3 \times 3 \times 3 \times 0.2 \times 0.2 \times 0.2 \times 0.2 = (3)^3(0.2)^4$

Evaluate each statement.

5. $(2)^3(3)^2$

Solution:
 $(2)^3(3)^2 = 2 \times 2 \times 2 \times 3 \times 3$
 $= 8 \times 9$
 $= 72$

6. $(1.2)^4$

Solution:
 $(1.2)^4 = (1.2)(1.2)(1.2)(1.2)$
 $= (1.44)(1.44)$
 $= 2.0736$

7. $\left(\frac{2}{3}\right)^3$

Solution:
 $\left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{27}$

8. $\left(3\frac{1}{2}\right)^2$

Solution:
 $\left(3\frac{1}{2}\right)^2 = \left(\frac{7}{2}\right)^2 = \left(\frac{7}{2}\right)\left(\frac{7}{2}\right) = \frac{49}{4}$

Express as a Single Power.

9. $7^3 \times 7^2$

$7^3 \times 7^2 = (7 \bullet 7 \bullet 7)(7 \bullet 7) = 7^5$
 OR
 $7^3 \times 7^2 = 7^{3+2} = 7^5$

10. $(-6)^3 \times (-6)^4$

$(-6)^3 \times (-6)^4 = (-6 \bullet -6 \bullet -6) \times (-6 \bullet -6 \bullet -6 \bullet -6) = (-6)^7$
 OR
 $(-6)^3 \times (-6)^4 = (-6)^{3+4} = (-6)^7$

11. $(5^2)^3$

$(5^2)^3 = 5^2 \times 5^2 \times 5^2 = 5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 = 5^6$
 OR
 $(5^2)^3 = 5^{2 \bullet 3} = 5^6$

12. $\frac{7^6}{7^2}$

$\frac{7^6}{7^2} = \frac{7 \bullet 7 \bullet 7 \bullet 7 \bullet 7 \bullet 7}{7 \bullet 7} = 7^4$
 OR
 $\frac{7^6}{7^2} = 7^{6-2} = 7^4$

Exercises 1.5.2

Evaluate

1. $(1.1)^2$

2. $(0.5)^3$

3. $(-1.5)^2$

4. $(-0.1)^3$

5. $\left(\frac{3}{5}\right)^3$

6. $(2)^3(0.2)^2$

7. $(-0.3)^3$

8. $\left(-\frac{1}{4}\right)^4$

9. $\left(\frac{5}{2}\right)^3\left(\frac{2}{5}\right)^2$

10. $(-2.1)^3(-.5)^2$

11. Express as a single power.

a. $6^4 \times 6^5 \times 6^3$

b. $7^2 \times 7^6 \times 7$

c. $4^{11} \times 4^4$

d. $2^5 \times 2^{15}$

e. $(0.5)^2 (0.5)^{11} (0.5)^3$

f. $\left(\frac{2}{3}\right)^5 \left(\frac{2}{3}\right)^7 \left(\frac{2}{3}\right)$

g. $\left(-\frac{1}{5}\right)^6 \left(-\frac{1}{5}\right) \left(-\frac{1}{5}\right)^{12}$

h. $\frac{6^9}{6^4}$

i. $\frac{10^4}{10^5}$

j. $\frac{(-5)^{12}}{(-5)^4}$

k. $\frac{4^5 \cdot 4^6}{4^7}$

l. $\frac{5^9 \cdot 5^3}{5^4}$

m. $\frac{(-3)^8 (-3)^8}{(-3)^3}$

n. $\frac{11^{10}}{11^2 \cdot 11^7}$

o. $\frac{(-2)^{11}}{(-2)(-2)^3}$