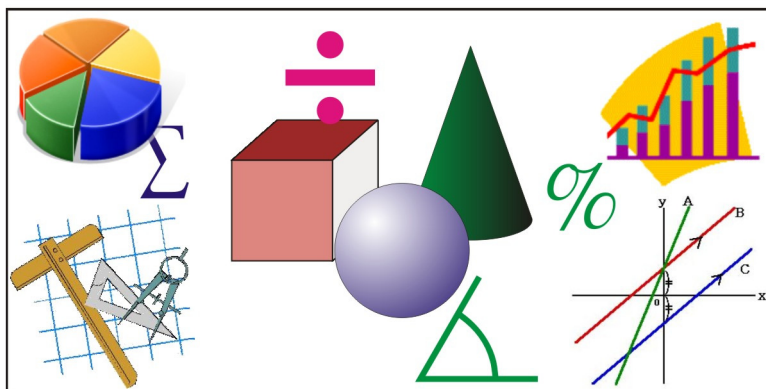


RAVEN'S LITERACY FOUNDATIONS LEVEL 5 MATHEMATICS

UP-GRADING PROGRAM

(Designed for the Western Provinces and the Territories)

STUDENT GUIDE AND RESOURCE BOOK



The Key to Student Success

One of a series of publications by Raven Research Associates
for Secondary and Elementary Mathematics

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Introduction

This book is intended to provide students taking **Literacy Foundations Level 5 Mathematics** with a practical resource designed to enhance success. It is linked to the provincial mathematics curriculum and designed by experienced teachers of mathematics to provide students with greater success in these courses. It includes the following features:

- Clear Descriptions of the Key Concepts
- Numerous Examples with Step-by-Step Solutions
- Many Practice Exercises to Reinforce Understanding and Application
- Review Exercises with a Range of Difficulty Levels
- All Answers which are listed at the Back of the Book
- Produced by Experienced Teachers of Mathematics
- Hand-in Exercises
- Attractively Bound and Formatted for Clarity and Ease of Access

The content areas listed in the Table of Contents, shown in the next page, are linked to the following Prescribed Learning Outcomes for Literacy Foundations Level 5 Mathematics. .

NUMBER

- | | |
|--|---|
| <p>A1 demonstrate an understanding of multiplication and division of fractions with like and unlike denominators concretely, pictorially, and symbolically</p> <p>A2 determine the relationship between positive repeating decimals and positive fractions (e.g., $1/3$) and positive terminating decimals and positive fractions (e.g., $1/8$)</p> <p>A3 compare and order positive fractions, decimals, and whole numbers using benchmarks (e.g., 0, $1/2$, 1)</p> <p>A4 solve problems involving the application of ratios and proportions</p> <p>A5 solve problems involving percentages and justify the solution</p> <p>A6 explain the process for addition, subtraction, multiplication, and division of integers concretely, pictorially, and symbolically</p> | <p>A7 demonstrate an understanding of powers as repeated multiplication and explain the meaning of the base, coefficient, and exponent in a power</p> <p>A8 write powers as the product of factors and explain their meaning</p> <p>A9 evaluate expressions involving powers with integral bases</p> <p>A10 demonstrate an understanding of exponent rules for the multiplication and division powers to solve problems</p> <p>A11 express a given number using scientific notation</p> <p>A12 simplify expressions, including exponents, using order of operations</p> |
|--|---|

PATTERNS AND RELATIONS

Patterns

- B1 represent algebraic expressions and equations in words
- B2 represent and describe patterns and relationships using graphs and a table of values

Variables and Equations

- B3 apply and explain how preservation of equality is used to solve equations

- B4 solve problems involving one- and two-step equations limited to equations of the form $\chi + a = b$, $a\chi = b$, and $a\chi + b = c$
- B5 distinguish between expressions and equations and explain the similarities and differences
- B6 evaluate an expression given the value of the variable

SHAPE AND SPACE

Measurement

- C1 explain the process for determining the circumference and area of a circle

3-D Objects and 2-D Shapes

- C2 calculate the volume of triangular prisms, cylinders, cones, and pyramids

Transformations

- C3 identify and plot points in the four quadrants of a Cartesian plane using ordered pairs

STATISTICS AND PROBABILITY

Data Analysis

- D1 read, interpret, and construct line graphs from a given data set

Chance and Uncertainty

- D2 express and interpret probabilities as ratios, fractions, or percents

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SAMPLE EXERCISE

1.5.2 Multiplication of Decimals

Estimating Products of Decimals Using Front End Estimation

Estimating Products of Decimals using Front-end Estimation

- We can **estimate** a product of a decimal and a whole number by first rounding the decimal to the nearest whole number. Then calculate the product of the resulting whole numbers.
e.g. 1. Estimate the product of 32.3 and 6. **Think** $32 \times 6 = 192$ (rounding 32.3 down to 32). The exact product is greater than 192 since we rounded 32.3 down.

2. Estimate the product of 49.7 and 12. **Think** $50 \times 12 = 600$ (rounding 49.7 up to 50). The exact product is less than 600 since we rounded 49.7 up.

Examples with Step-by-Step Solutions

Use front-end estimation to find the approximate value of each product.

- | | |
|------------------------|---|
| a. 41.2×7 | <ul style="list-style-type: none"> ▪ $41 \times 7 = 287$ ▪ <i>the exact product is greater than 287</i> |
| b. 141.3×72 | <ul style="list-style-type: none"> ▪ $141 \times 72 = 10\ 152$ ▪ <i>the exact product is greater than 10 152</i> |
| c. 45×6.8 | <ul style="list-style-type: none"> ▪ $45 \times 7 = 315$ ▪ <i>the exact product is less than 315</i> |
| d. 348.7×33.9 | <ul style="list-style-type: none"> ▪ $349 \times 34 = 11\ 866$ ▪ <i>the exact product is less than 11 866</i> |

Products of Decimals Using Expansion of Numbers

Finding Products of Decimals using the Expansion of Numbers

- We can find the product of a whole number and a decimal number by first writing the decimal number as an expanded number. Next separate the whole number part and the decimal part. Then multiply each part by the whole number and add the result.

Examples

1. 55.2×6 . **Think** $(55 + 0.2) \times 6 = 55 \times 6 + 0.2 \times 6 = 330 + 1.2 = 331.2$
(i) expand (ii) multiply parts (iii) add result
2. 45.5×8 **Think** $(45 + 0.5) \times 8 = 45 \times 8 + 0.5 \times 8 = 360 + 4 = 364$

Examples with Step-by-Step Solutions

Use the expansion of one factor to find each product.

Solution

- a. 78.2×5
- $(78 + 0.2)5$
 - $78 \times 5 + 0.2 \times 5 = 390 + 1 = 391$
- b. 150.3×7
- $(150 + 0.3)7$
 - $150 \times 7 + 0.3 \times 7 = 1050 + 2.1 = 1052.1$
- c. 259.8×6
- $(259 + 0.8)6$
 - $259 \times 6 + 0.8 \times 6 = 1554 + 4.8 = 1558.8$
- d. 55.4×3
- $(55 + 0.4)3$
 - $55 \times 3 + 0.4 \times 3 = 165 + 1.2 = 166.2$

Multiplication of Decimals Using Rules

Multiplication of Decimals using Rules

- First, multiply the numbers as you would whole numbers.
- Next, when multiplying a decimal or mixed decimal by a whole number, count off the same number of decimal places in the answer as there are in the factors being multiplied.

e.g. 1. $8.25 \leftarrow$ two decimal places 2. $9.3 \leftarrow$ one decimal place
 $\begin{array}{r} 8.25 \\ \times 5 \\ \hline 41.25 \end{array} \leftarrow$ no decimal places $\begin{array}{r} 9.3 \\ \times 3.5 \\ \hline 32.55 \end{array} \leftarrow$ one decimal place
 $41.25 \leftarrow$ total two decimal places $32.55 \leftarrow$ total two decimal places

Examples with Step-by-Step Solutions

Use the rules for multiplying with decimals to find each product.

- a. 51.3×4.1
- $$\begin{array}{r} 51.3 \quad \leftarrow \text{one decimal place} \\ \times 4.1 \quad \leftarrow \text{one decimal place} \\ \hline 210.33 \quad \leftarrow \text{two decimal places} \end{array}$$
- b. 345.1×8
- $$\begin{array}{r} 345.1 \quad \leftarrow \text{one decimal place} \\ \times 8 \\ \hline 2760.8 \quad \leftarrow \text{one decimal place} \end{array}$$
- c. 22.35×6.5
- $$\begin{array}{r} 22.35 \quad \leftarrow \text{two decimal places} \\ \times 6.5 \quad \leftarrow \text{one decimal place} \\ \hline 145.275 \quad \leftarrow \text{three decimal places} \end{array}$$
- d. 254.21×27.31
- $$\begin{array}{r} 254.21 \quad \leftarrow \text{two decimal places} \\ \times 27.31 \quad \leftarrow \text{two decimal places} \\ \hline 6942.4751 \quad \leftarrow \text{four decimal places} \end{array}$$
- e. 42.02×3.012
- $$\begin{array}{r} 42.02 \quad \leftarrow \text{two decimal places} \\ \times 3.012 \quad \leftarrow \text{three decimal places} \\ \hline 126.56424 \quad \leftarrow \text{five decimal places} \end{array}$$
-

Problems Involving Multiplication of Decimals

- Next we are going to solve word problems involving multiplication of decimals. To solve these word problems it is suggested that you use the following steps.

Steps to Solve a Word Problem

- | | |
|--|---|
| 1. What is the question? | 1. Read the question carefully to identify what is asked for. |
| 2. What are you told about the question? | 2. Look at the information to see what it tells about the question. |
| 3. Write a mathematical statement | 3. Write a mathematical sentence that describes the information. |
| 4. Find the value of the statement | 4. Find a number value that makes the statement true. |
| 5. Check the answer. | 5. Check to make sure the value works. |
| 6. Write a concluding word statement. | 6. Write a concluding statement that answers the question |

Example



Earl cycled for 15.3 km each day for 20 days. How far did cycle altogether during that time?

- | | |
|--|---|
| 1. What is the question? | 1. How far did Earl cycle altogether? |
| 2. What are you told about the question? | 2. He cycled a distance of 15.3 km each day for 20 days |
| 3. Write a mathematical statement | 3. 15.3×20 equals the total distance |
| 4. Find the value of the statement | 4. $Total = 15.3 \times 20 = 306$ |
| 5. Check the answer. | 5. $306 \div 20 = 15.3 \checkmark$ |
| 6. Write a concluding statement. | 6. He cycled a total distance of 306 km in 20 days. |

Examples with Step-by-Step Solutions

1. A package of flour has a mass of 1.52 kg. What mass would 12 packages of flour have?

Solution

- | | |
|--|---|
| 1. What is the question? | 1. <i>What is the mass of 12 packages of flour?</i> |
| 2. What are you told about the question? | 2. <i>Each package of flour has a mass of 1.52 kg.</i> |
| 3. Write a mathematical statement | 3. <i>The total mass is 1.52×12</i> |
| 4. Find the value of the statement | 4. <i>$1.52 \times 12 = 18.24$</i> |
| 5. Check the answer. | 5. <i>$18.24 \div 12 = 1.52$</i> |
| 6. Write a concluding statement. | 6. <i>The mass of 12 packages of flour is 18.24 kg.</i> |
2. Bill picked 5 baskets of strawberries. Two of the baskets had a mass of 2.3 kg each and the other three had a mass of 2.5 kg each. How many kilograms of strawberries did he pick altogether?

Solution

- | | |
|--|--|
| 1. What is the question? | 1. <i>How many kilograms of strawberries did Bill pick altogether?</i> |
| 2. What are you told about the question? | 2. <i>He picked 2 baskets with a mass of 2.3 kg each and 3 baskets with a mass of 2.5 kg each.</i> |
| 3. Write a mathematical statement | 3. <i>Total mass is $2 \times 2.3 + 3 \times 2.5$</i> |
| 4. Find the value of the statement | 4. <i>$2 \times 2.3 + 3 \times 2.5 = 4.6 + 7.5 = 12.1$</i> |
| 5. Check the answer. | 5. <i>$2.3 + 2.3 + 2.5 + 2.5 + 2.5 = 12.1$</i> |
| 6. Write a concluding statement. | 6. <i>Bill picked 12.1 kg of strawberries altogether.</i> |
-

Exercises 1.5.2

1. Use any method to find the exact value of each product.

a.
$$\begin{array}{r} 28.5 \\ \times 3 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 42.2 \\ \times 4.1 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 50.7 \\ \times 2.3 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 45.56 \\ \times 2.2 \\ \hline \end{array}$$

e.
$$\begin{array}{r} 42.01 \\ \times 3.1 \\ \hline \end{array}$$

f.
$$\begin{array}{r} 450.6 \\ \times 2.5 \\ \hline \end{array}$$

g.
$$\begin{array}{r} 6.03 \\ \times 7.1 \\ \hline \end{array}$$

h.
$$\begin{array}{r} 800.6 \\ \times 20.5 \\ \hline \end{array}$$

i.
$$\begin{array}{r} 18.005 \\ \times 1.2 \\ \hline \end{array}$$

j. 3.01×33.5

k. 35.2×7.4

l. 8.1×2.03

2. Jo ran for 8.6 km each day for 35 days. How far did she run altogether during that time?

3. How much would 5 kg of steak cost if the price was \$8.75 per kilogram?

4. In order to complete work on the living room, John needed two pieces of baseboard measuring 6.5 m each and two pieces measuring 4.2 m each. How much baseboard did he need altogether?

5. Liz bought three candy bars for \$0.85 each and 2 pens for \$1.95 each. How much did she spend altogether?

6. Sue purchased 5 sacks of rice for her restaurant. If each sack had a mass of 20.52 kg, how many kilograms of rice did she buy?

7. The telephone costs for a long distance call from Vancouver to Toronto is \$3.95 for the first three minutes and \$0.65 per minute for any additional time. At this rate, how much would a 10-minute call cost?

8. The mass of a nickel is about 5 grams. What would the mass of \$6.50 worth of nickels be?

9. Charlene wanted to purchase 9 audio tapes. She could purchase them in packages of 3 for \$8.85 per package or else individually for \$3.25 each. How much would she save by purchasing them in packages of 3 rather than individually?

10. Tara paid \$2.55 for one dozen eggs and \$0.25 each for an additional three. How much change should she get from a 20-dollar bill?